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Marcel Boyer
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Georges Dionne@

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Résumé

Il existe plusieurs situations économiques pour lesquelles les agents ont à choisir entre des variations de la probabilité (p) ou du montant de perte (ℓ). L'analyse des variations de risque associées à ce choix n'est pas explicite dans la littérature économique. Isoler les variations de risque est important pour trois raisons : premièrement, cela permet de calculer les effets de ces variations sur le bien-être des individus ayant des attitudes différentes face au risque, deuxièmement, cela permet de calculer la compensation monétaire entre les groupes d'individus ayant des attitudes différentes face au risque et, troisièmement, cela permet de clarifier les questions empiriques. Dans ce texte nous isolons les variations de risque, associées à des changements équivalents entre p et ℓ, et nous analysons l'effet de ces variations sur le bien-être des individus. Pour ce faire nous utilisons la définition de l'accroissement de risque proposée par Rothschild et Stiglitz (1970). Nous démontrons qu'un individu ayant de l'aversion au risque préfère toujours une baisse dans ℓ à une réduction équivalente dans p. Ce résultat est ensuite appliqué dans trois cas. D'abord nous considérons le cas dans lequel il n'y a pas d'activité gouvernementale pour affecter la probabilité ou le montant de perte (activités d'autoprotection vs activités d'autoassurance par exemple); puis nous considérons le cas dans lequel la probabilité ou le montant de perte peuvent être affectés seulement par des actions gouvernementales (probabilité vs montant d'amende); et finalement nous considérons le cas

général où les actions individuelles et gouvernementales peuvent affecter la probabilité et le montant de perte (probabilité d'accident vs gravité des accidents). Nous présentons aussi une méthode pour calculer la compensation optimale à payer aux individus plus riscophobes, lorsque les individus moins riscophobes choisissent les niveaux de p et ℓ pour la société.

Abstract

There are many economic situations in which the agents have to choose between shifting the probability (p) or changing the magnitude of loss (ℓ). The analysis of the variations of risk associated with these activities is not explicit in the economic literature. Isolating the variations of risk is important for three reasons : first, it permits to calculate the effects of these variations on the welfare of individuals having different attitudes towards risk, second, it permits to calculate the optimal compensation between groups of individuals having different attitudes towards risk and, third, it permits to clarify the issues for empirical studies. In this paper we isolate the variations of risk, caused by equivalent changes in p and ℓ, and show how these variations affect the welfare of individuals. This is done using the definition of increasing risk proposed by Rothschild and Stiglitz [1970]. We show that risk averse individuals always prefer a decrease in ℓ to an equivalent reduction in p. This result is applied in three cases of increasing complexity. First we consider the case where there are no government activities but only individual activities affecting the probability and magnitude of loss (v.g. self-insurance vs self-protection). Then we consider the case where changes in the probability and magnitude of loss can be achieved only through governmental actions (v.g. probability vs magnitude of fine). Finally we consider the general case where both individual activities and governmental activities are available to change the probability and magnitude of loss (v.g. accident probability vs severity). We present also a method to calculate the optimal compensation to pay to the more risk averse individuals when the less risk averse individuals choose the levels of p and ℓ for the society.

Abstract

There are many economic situations in which the agents have to choose between shifting the probability (p) or changing the magnitude of loss (ℓ). The analysis of the variations of risk associated with these activities is not explicit in the economic literature. Isolating the variations of risk is important for three reasons : first, it permits to calculate the effects of these variations on the welfare of individuals having different attitudes towards risk, second, it permits to calculate the optimal compensation between groups of individuals having different attitudes towards risk and, third, it permits to clarify the issues for empirical studies. In this paper we isolate the variations of risk, caused by equivalent changes in p and ℓ, and show how these variations affect the welfare of individuals. This is done using the definition of increasing risk proposed by Rothschild and Stiglitz [1970]. We show that risk averse individuals always prefer a decrease in ℓ to an equivalent reduction in p. This result is applied in three cases of increasing complexity. First we consider the case where there are no government activities but only individual activities affecting the probability and magnitude of loss (v.g. self-insurance vs self-protection). Then we consider the case where changes in the probability and magnitude of loss can be achieved only through governmental actions (v.g. probability vs magnitude of fine). Finally we consider the general case where both individual activities and governmental activities are available to change the probability and magnitude of loss (v.g. accident probability vs severity). We present also a method to calculate the optimal compensation to pay to the more risk averse individuals when the less risk averse individuals choose the levels of p and ℓ for the society.

Introduction and Summary

There are many economic situations in which the agents have to choose between shifting the probability or changing the magnitude of loss. As examples, individuals have to decide whether to use self-protection or self-insurance activities, with and without insurance, in order to protect themselves against economic losses associated to accident or illness. The governments have to establish the magnitude and probability of fines in order to motivate individuals to reduce those activities which impose social costs such as double parking and speeding. Moreover considering events such as inflation and war, the governments have to decide on activities, unavailable on an individual basis, in order to reduce the welfare losses associated to the occurrence of those bad events.

The analysis of the variation of risk associated with these activities is not explicit in the economic literature. Of course, many authors[1] have considered the variation of risk and its effects on resources allocation, but have not isolated the effects of the variation of risk from other effects[2]. Isolating the variation of risk is important for three reasons : first, it permits to calculate the effects of these variations on the welfare of individuals having different attitudes towards risk, second, it

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[1]The three main papers in relation to the specific examples mentioned above are those of Ehrlich and Becker [1972], Peltzman [1975], Polinsky and Shavell [1979].

[2]Our comment concerns only situations in which it is possible to isolate the variation of risk.

permits to calculate the optimal compensation between groups of individuals having different attitudes towards risk and, third, it permits to clarify the issues for empirical studies.

For example, does an individual prefer self-protection to self-insurance; does he prefer reduced highway speed to increased safety devices on automobiles; does he choose one or the other because it costs less for him, because the reduction of risk is greater or because he is less risk averse than other individuals (assuming that the efficiency of both policies are the same)?

The object of this paper is to isolate the effects of the variations of risk on the welfare of individuals in order to answer the preceding questions. We will analyze three cases of increasing complexity. After introducing (section I) the notation and the basic model, we proceed with the case (section II) where there are no government activities but only individual activities affecting the probability and magnitude of loss. Then we consider the case (section III) where changes in the probability and magnitude of loss can be achieved only through governmental actions. Finally we consider the general case (section IV) where both individual activities and governmental activities are available to change the probability and magnitude of loss.

In these three cases the methodology used shall be the same. We use the definition of increasing risk proposed by Rothschild and Stiglitz [1970]. They have shown that, confronted with two situations A and B such that B can be obtained by applying a mean preserving spread

to A, a risk averse individual always prefers A to B. Using this result of Rothschild and Stiglitz, we will identify, in the many examples analyzed in this paper, which activity is the most risky and then, isolating the effects of risk from others, we will identify which activities risk averse individuals prefer.

The main results of the paper are the following :

- In the first case (individuals' actions only) it is shown that risk averse individuals always prefer self-insurance to self-protection since the former is more efficient in terms of reduction of risks. This result is obtained under the assumptions that both activities are equally efficient to reduce the expected loss and that both activities have the same cost function. Moreover, under the same assumptions, risk averse individuals are indifferent between market insurance and self-insurance when the probability of loss is constant, since both are equally efficient in terms of reduction of risk. However, when the probability of loss is not constant, risk averse individuals prefer self-insurance to market insurance under perfect information. This last result is due to the fact that under market insurance, risk averse individuals have some incentive to use self-protection activities in order to reduce their insurance premium. This implies that self-protection is greater under market insurance than under self-insurance for all levels of insurance, and that, for a same variation of the expected loss, self-insurance is greater than market insurance. Combining the preceding results permits us to show that the reduction of risk is greater under self-insurance.

- We also show that the benefits of using the more efficient activity in terms of reduction of risk, increase with risk aversion. A direct corollary is that risk neutral individuals are indifferent between those activities in term of variation of risk.
- Under imperfect information on both self-protection activities and the degree of risk aversion, a simple model of partial insurance coverage is used with linear pricing. It is shown that risk averse individuals choose the maximum amount of insurance available if the less risk averse buy this amount and that risk averse individuals prefer to use self-insurance rather than self-protection to complete market insurance. This result has important implications for the study of moral hazard since self-insurance is rarely considered in the economic literature on moral hazard.
- Considering the second case, i.e. government intervention without individuals' activities, we develop a model similar to the model of Polinsky and Shavell [1979] who analyzed the determination of the optimal[1] trade-off between the probability and the magnitude of fine. They showed that, if individuals are risk neutral, the optimal probability is equal to the threshold probability of those who gain the least in having activities which impose social costs and the optimal fine is equal to the individual's wealth. We show here that, when it is costless to catch individuals who engage in double parking (as example), risk neutral individuals are indifferent between any combination of the probability (greater than the threshold probability of those who gain the least) and fines (smaller

[1]Here optimal trade-off means Pareto optimal or socially optimal trade-off.

than the individual's wealth) which corresponds to the same expected tax benefits than the combination proposed by Polinsky and Shavell. Moreover they showed that, under risk aversion, the optimal fine equals the private gain of those who engage in double parking and the optimal probability is equal to one (when the cost is sufficiently low). Extending the analysis we show that this result remains true for higher cost functions as risk aversion increases. Moreover we compute the optimal compensation to pay to the highest risk averse group when the less risk averse individuals choose the optimal trade-off for the society by majority rule.

- Tackling the general case corresponding to both types of actions by both the individual agent and the government (auto safety regulation is a good example of such a case), we show that if individuals' reactions to the variation of risk associated with government policies are taken into account, the results of these policies on individuals' utilities (still in terms of variation of risk) are now ambiguous under risk aversion. This is true since private reactions may have effects on the variation of risk opposite to those of the government policies. However when the direct effects of government policies are larger than those of indirect private reactions, we show that risk averse individuals always prefer the policy that reduces the amount of loss to the one reducing the probability of loss.

I- Notation and Assumptions of the Basic Model

In this section of the paper, the assumptions and the notation of the basic model are presented. The specific assumptions will be introduced when needed in the subsequent sections.

Let's assume that all individuals are identical in all respect except for their degree of risk aversion. There are two classes of individuals in the economy considered : individuals i, in the class I, are less risk averse than individuals j in the class J. However all individuals are identical in each class of risk aversion. Let $U_k(S)$ be the utility of wealth S of consumer k, k = i, j, with $U_k'(S) > 0$ and $U_k''(S) \leq 0$. In the Arrow-Pratt sense[1] we have

$$-\frac{U_i''(S)}{U_i'(S)} \leq -\frac{U_j''(S)}{U_j'(S)} \quad \forall S \quad \text{and for all } i \in I \text{ and } j \in J$$

Each individual k faces two possible States of the world : one corresponding to no monetary loss and another corresponding to a monetary loss, $\ell(y_k)$, a function of y_k representing the self-insurance activities of the individual. We assume that $\ell'(y_k) < 0$ and $\ell''(y_k) > 0$. The probability of incurring a loss is $p(x_k)$, a function of x_k representing the self-protection activities[2] of individual k. We assume that $p'(x_k) < 0$ and $p''(x_k) > 0$.

Each individual is assumed to maximize the expected value Z of his utility, a function of wealth S and of his activities x and y and of q, the amount of market insurance he decides to buy.

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[1]Since it is explicitly assumed that there is only one gamble, the classical Arrow-Pratt measure of absolute risk aversion is enough to compare individuals (Arrow [1971], Pratt [1964]). For the case of many gambles see Ross [1981].

[2]The concepts of self-insurance and self-protection were introduced by Erhlich and Becker [1972].

When the distinction between individuals is not necessary, we drop the subscript k.

(1) $$Z(q, x, y) = (1 - p(x))U(\bar{S} - P(q) - C(x, y)) + p(x)U(\bar{S} - P(q) - \ell(y) + q - C(x, y))$$

where $\bar{S}$ = initial wealth

$P(q)$ = the insurance premium (by assumption, there is no transaction costs, no profit and the insurers are risk neutral). When $P(q)$ is linear, we use πq where π is the price of insurance.

$C(x, y)$ = cost function in monetary terms of activities x and y; we assume $C_x > 0$, $C_y > 0$, $C_{xx} > 0$, $C_{yy} > 0$ and $C_{xy} = 0$ for simplicity.

II- Self-insurance, Self-protection and Market Insurance

IIA) Market insurance is not available

Self-insurance and self-protection are presented as substitutes in the economic literature : "that is to say, the availability of self-insurance would discourage self-protection and vice-versa. Moreover, technological progress in the provision of one would tend to discourage the other" (Ehrlich and Becker [1972]). In this section of the paper we consider that three factors affect the choice between those two activities when market insurance is not present : 1) the efficiency to reduce expected loss, 2) the cost of each activity and 3) the degree of risk aversion of the consumer.

Proposition I : Assume that the average loss is changed by the same amount using given variations in either self-insurance or self-protection activities with $q \equiv 0$ and assume that these variations imply the same costs; then a risk averse consumer always prefer self-insurance to self-protection since the former is more efficient in terms of reduction of risk.

Proof : Without market insurance, the expected utility function of the individual is equal to :

$$(2) \quad Z(0, x, y) = (1 - p(x))U(\bar{S} - C(x, y)) + p(x)U(\bar{S} - \ell(y) - C(x, y))$$

Taking the differential of (2) with respect to x and y gives :

$$(3) \begin{cases} 3.a) \ \partial Z = \{p'[U(\bar{S} - \ell(y) - C(x, y)) - U(\bar{S} - C(x, y))] - C_x EU'\}\partial x \\ 3.b) \ \partial Z = \{- p(x)U'(S - \ell(y) - C(x, y))\ell' - C_y EU'\}\partial y \end{cases}$$

where $EU' \equiv (1 - p(x))U'(\bar{S} - C(x, y)) + p(x)U'(\bar{S} - \ell(y) - C(x, y))$

and $p' \equiv \frac{dp}{dx}$ and $\ell' \equiv \frac{d\ell}{dy}$

Assuming that the variation of the expected loss is the same using both activities, means that the variation of the expected wealth ($\mu = \bar{S} - p(x)\ \ell(y)$) is the same,

$$(4) \begin{cases} 4.a) \ \partial\mu = [p'\ell(y)]\partial x \\ 4.b) \ \partial\mu = [p(x)\ell']\partial y \end{cases}$$

implying

$$(5) \qquad \partial x = \left\{\frac{p(x)\ell'}{p'\ell(y)}\right\} \partial y.$$

Now assuming that the cost function is such that these variations have the same cost in utility terms, that is

$$(6) \qquad C_y \, EU' \, \partial y = C_x EU' \left\{ \frac{p(x)\ell'}{p'\ell(y)} \right\} \partial y$$

then from (3) the variation in self-insurance activities ∂y is prefered to the equivalent (satisfying (5) and (6)) variation in self-protection activities if and only if, substituting (5) and (6) in (3),

$$(7) \qquad p'[U(\bar{S} - \ell(y) - C(x, y)) - U(\bar{S} - C(x,y))] \left\{ \frac{p(x)\ell'}{p'\ell(y)} \right\} \partial y < - p(x)U'(\bar{S} - \ell(y) - C(x, y))\ell'\partial y$$

that is if and only if

$$(8) \quad U(\bar{S} - \ell(y) - C(x, y)) - U(\bar{S} - C(x, y)) > - U'(\bar{S} - \ell(y) - C(x, y))\ell(y)$$

Using a Taylor's expansion, the left hand side of (8) may be approximated by

$$(9) \quad - [U'(\bar{S} - \ell(y) - C(x, y))\ell(y) + U''(\bar{S} - \ell(y) - C(x, y)) \frac{\ell(y)^2}{2} + R]$$

Assuming that R is negligeable, (8) becomes

$$(10) \quad U'(\bar{S} - \ell(y) - C(x, y))\ell(y) + U''(\bar{S} - \ell(y) - C(x, y)) \frac{\ell(y)^2}{2} < U'(\bar{S} - \ell(y) - C(x, y))\ell(y)$$

and since $U''(S) < 0$ under risk aversion, then the right hand side of (10) is always greater than the left hand side, which means that self-insurance is prefered to self-protection by risk averse consumers.

Q.E.D.

This result is due to the fact that there is a mean preserving spread (Rothschild and Stiglitz [1970]) between the two resulting wealth distributions. This is true since the following conditions defining a mean preserving spread are fulfilled : defining F(S) and G(S) as the c.d.f.'s of S corresponding respectively to the distributions of S resulting from the variations ∂x and ∂y in self-protection and self-insurance[1] activities, we have :

$$(11)\quad \begin{cases} A) & \int_0^{\bar{S}} [F(S) - G(S)]dS = 0 \\ B) & \exists\ \hat{S} \text{ such that } F(S) - G(S) > 0 \text{ when } S < \hat{S} \\ & \qquad\qquad F(S) - G(S) \leq 0 \text{ when } S \geq \hat{S} \\ C) & \int_0^{\tilde{S}} [F(S) - G(S)]ds > 0 \text{ for } 0 \leq \tilde{S} \leq \bar{S} \end{cases}$$

Under risk neutrality it follows immediately from Proposition I that :

Corollary I : Under the same assumptions as those in Proposition I, a risk neutral consumer is indifferent between self-insurance and self-protection.

Proof : The proof is straightforward from (10) since under risk neutrality

$$U''(S) = 0$$

Q.E.D.

[1] For a detailed analysis of a simple case of mean preserving spread see Boyer and Dionne (1982).

Proposition II : Under the same assumptions as those in Proposition I, the net benefit associated to self-insurance increases with risk aversion.

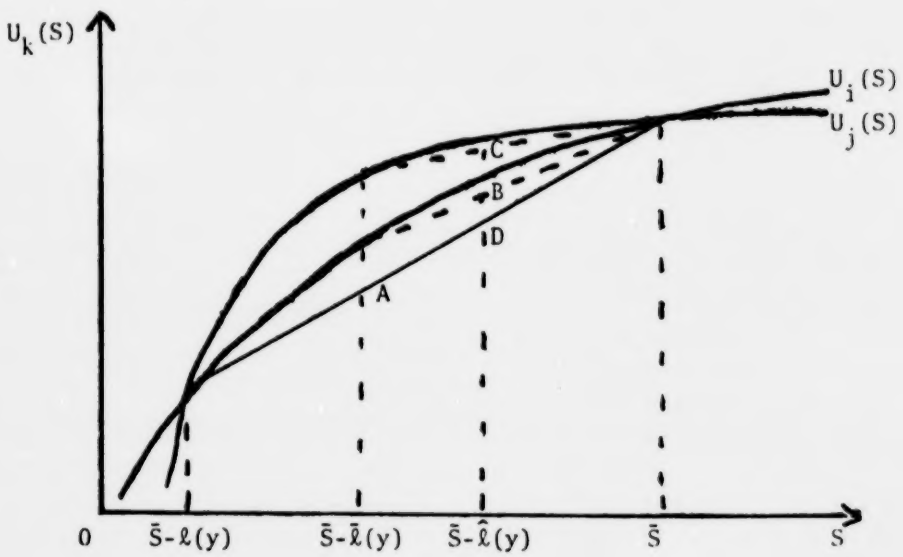
We can write (10) as follows :

$$(12) \qquad 1 + \frac{U''(\bar{S} - \ell(y) - C(x,y))}{U'(\bar{S} - \ell(y) - C(x,y))} \left(\frac{\ell(y)}{2}\right) < 1$$

An increase in risk aversion makes the left hand side smaller.

Q.E.D.

We can illustrate graphically the above propositions. Consider two typical individuals i and j with, as assumed in section I, i being less risk averse than j, and suppose that $U_i(\bar{S}) = U_j(\bar{S})$ and $U_i(\bar{S} - \ell(y)) = U_j(\bar{S} - \ell(y))$. Therefore we have :



(Figure 1)

where $\bar{S} - \bar{\ell}(y)$ is the mean of the initial situation and also the level of wealth after self-insurance. $\bar{S} - \hat{\ell}(y)$ is the mean of the new situation after self-insurance and self-protection. A is the initial level of expected utility of individuals of type i and j. D is the level of expected utility after self-protection for individuals of types i and j. B is the level of expected utility of individual i after self-insurance and C is the corresponding level for individual j. From Proposition I we have that B and C are greater than D and, from Proposition II, we have that $C > B$. Finally, under risk neutrality D is also the level of expected utility after self-insurance which implies, from Corollary I, that self-insurance and self-protection are equivalent for risk neutral individuals.

IIB) Market insurance is available

Market insurance and self-insurance are known as close substitutes [Ehrlich and Becker 1972] since the demand for self-insurance increases as the price of insurance increases, the probability of loss being constant. It is also well known that, under market insurance and asymmetrical information on self-protection, moral hazard is potentially present. Many insurance strategies[1] can be used in order to reduce the welfare loss associated to moral hazard. However those strategies are used under the assumption that there exist only one class of risk aversion in the economy. When there are more than one class of risk aversion, the problem is much more complicated especially if we assume that the insurers cannot observe without cost to what class of risk aversion

[1]Pauly [1974], Helpman and Laffont [1975], Shavell [1979], Dionne [1981].

each individual belongs. Since the object of this paper is not to characterize the optimal insurance policy with moral hazard and many classes of risk aversion, we will assume that the equilibrium price (linear) of insurance $\bar{\pi}$ exists and that market insurance is cheaper than self-insurance. Moreover we will assume that $\bar{q}(\bar{q} < \ell)$ is the maximum amount of insurance available under moral hazard in order to make incentive for self-protection and self-insurance assuming that $Z(\bar{q}, x, y) \geq Z(\ell, 0, 0)$ where $Z(\ell, 0, 0)$ is the expected utility level under full insurance and moral hazard.

In this section of the paper, we first show that the optimal choice between market insurance and self-insurance does not depend on risk aversion when the probability of loss is constant. That is, for a same variation of the average loss (no mean preserving spread here), the choice between self-insurance and market insurance only depends on the cost of each kind of insurance. This result is closed to the result of Ehrlich and Becker [1972] who have shown that if market insurance is available at a fair price, a risk averse consumer would act as if he is risk neutral and choose the amount of self-insurance that maximizes his expected income. Therefore if the price of market insurance is less than the shadow price of self-insurance for all levels of q and y, market insurance will be chosen whatever the degree of risk aversion.

Then we analyze the case when the probability of loss is not constant but can be observed without cost. In that case self-protection decreases more rapidly under self-insurance than under market insurance since individuals have incentive to keep their insurance premium low under

market insurance. We will show that for a same variation of the average loss and for a same variation of costs (associated to this variation of the average loss) risk averse individuals prefer self-insurance to market insurance since the reduction of risk is larger. This is true, since for a same variation of the expected loss, the variation of self-insurance has to be larger than the variation of market insurance because self-protection is larger under market insurance. But risk neutral individuals are indifferent between both kinds of insurance.

Finally we study the case under asymmetrical information by considering two levels of risk aversion. We will show that under the assumptions presented above, both types of agents will buy the maximal amount of insurance available if the less risk averse type buys that amount and, then, by extension of the preceding results, we will show that risk averse individuals prefer to use self-insurance rather than self-protection to complete their protection against risk.

Proposition III : Assume that the average net loss is affected by the same amount using variations ∂y and ∂q of self-insurance and market insurance, that these variations imply the same cost and that the probability of loss is constant. Then a risk averse individual is indifferent between the two variations.

Proof : Since $p(x)$ is constant, we can put $x \equiv 0$; the individual maximizes $Z(q, 0, y)$. Defining $U(1) = U(\bar{S} - \pi q - C(0, y))$ and $U(2) = U(\bar{S} - \pi q - \ell(y) + q - C(0, y))$ we have

$$(13) \qquad \partial Z(q, 0, y) = \{- \pi EU' + pU'(2)\}\partial q$$

(14) $\partial Z(q, 0, y) = \{- p\ell'U'(2) - C_y EU'\}\partial y$

Assuming that both variations of insurance are equivalent in terms of expected net loss and have the same cost means that :

(15) $\partial q = - \ell'\partial y$

and

(16) $C_y EU'\partial y = \pi EU'\partial q$

Then the individual will be indifferent between ∂y and ∂q satisfying (15) and (16) if and only if

$$pU'(2)\ (- \ell'\partial y) = -p\ell'U'(2)\partial y$$

which is always verified.

Q.E.D.

Proposition III states that the individual is indifferent between self-insurance and market insurance when the probability of loss cannot be changed by self-protection activities. The next proposition deals with a probability function $p(x)$ with $p'(x) < 0$; we want to compare a variation ∂y in self-insurance to a variation ∂q in market insurance considering that the individual will adjust his self-protection activities level x and therefore change the probability of loss. In adjusting his self-protection activities, the individual will consider two effects : the availability of insurance - either self-insurance or market insurance - is likely to decrease the need for self-protection because the difference between the wealth in the good and the bad state

is reduced and second, when the premium is a decreasing function of x, the availability of market insurance encourage a higher x. We will assume here that insurance decreases self-protection and that the premium effect makes the reduction in x from an increase in market insurance q smaller than from a decrease in the loss $\ell(y)$. That is

$$x_q = -\frac{\partial x}{\partial(\ell-q)} + \text{premium effect} < 0$$

$$x_\ell = \frac{\partial x}{\partial(\ell-q)} > 0$$

and

$$-x_\ell < x_q < 0$$

Proposition IV : Assume that the probability of loss is observable and decreases with self-protection activities x and that $-x_\ell < x_q < 0$. Assume that ∂q and ∂y have the same effect on expected net loss $[-\ell(y) + q]p(x)$ and are equally costly. Then a risk averse individual will always prefer the increase in self-insurance activities ∂y to the equivalent - in terms of costs and expected net loss - variation in market insurance ∂q.

Proof : Under the assumption that the probability of loss varies with self-protection activities x and is observable, we have both p and π as fonction of x. The expected utility is given by

$$(17)\quad Z(q, x, y) = (1-p(x))U(\bar{S} - \pi(x)q - C(x, y)) + p(x)U(\bar{S} - \pi(x)q - \ell(y) + q - C(x, y))$$

taking account of the impact of ∂q and ∂y on x and letting

$EU' = (1-p(x))U'(1) + p(x)U'(2)$, we have at a point (q, x, y)

$$(18)\quad \partial Z(q, x, y) = \{p(x)U'(2) - \pi(x)EU' + x_q[p'(U(2) - U(1)) - \pi' q\, EU' - C_x\, EU']\}\partial q$$

$$(19)\quad \partial Z(q, x, y) = \{-\, p(x)\ell'U'(2) - C_y EU' + x_y[p'(U(2) - U(1)) - \pi' q\, EU' - C_y\, EU']\}\partial y$$

The assumption that ∂q and ∂y are such that they have the same impact on expected net loss implies that

$$(20)\quad [-\, p(x) + p'x_q(\ell(y) - q)]\partial q = [p(x)\ell' + p'x_y(\ell(y) - q)]\partial y < 0$$

and the assumption that ∂q and ∂y are equally costly implies that

$$(21)\quad [\pi(x) + x_q(\pi' q + C_x)]\partial q = [C_y + x_y(\pi' q + C_x)]\partial y > 0$$

Substituting (20) and (21) in (18) and (19) we get that the individual will prefer ∂y to ∂q if and only if

$$(21')\quad \{p(x)U'(2) + x_q\, p'(U(2) - U(1))\} \left\{\frac{p(x)\ell' + p'x_y(\ell(y)-q)}{-p(x)+p'x_q(\ell(y)-q)}\right\} <$$

$$\{-\, p(x)\ell'U'(2) + x_y p'(U(2) - U(1))\}$$

Rearranging the terms and using (20) we can rewrite (21') as

$$[x_y + x_q\ell'][U'(2)\,(\ell(y) - q)) + (U(2) - U(1))] < 0$$

or, using a Taylor's expansion of [U(2) - U(1)], as

$$(22) \quad [x_y + x_q \ell'] \left[- U''(2) \frac{(\ell(y) - q)^2}{2} \right] < 0$$

But $[x_y + x_q \ell'] < 0$ since $x_y = x_\ell \ell'$ and by assumption $(- x_y < x_q < 0)$; and so (22) is verified under risk aversion.

Q.E.D.

Corollary II : Under the assumptions of Proposition IV, the cumulative distribution of S, say Q(S), obtained after the change in market insurance ∂q, can be obtained by adding a mean preserving spread F(S) to H(S), the cumulative distribution of S obtained after the equivalent change in self-insurance ∂y.

Proof : Let T(S) be the cumulative distribution of S before any change in q and y. In each case, S can take only two values since there are only two possible states. T(S) is the following

$$T(0) = 0$$

$$T(\bar{S} - \pi(x)q - C(x, y) - \ell(y) + q) = T(S_2) = p(x)$$

$$T(\bar{S} - \pi(x)q - C(x, y)) = T(S_1) = 1$$

Considering the adjustment in x following $dq = \hat{q} - q$ and $dy = \tilde{y} - y$ to be respectively $\hat{x} - x$ and $\tilde{x} - x$, we can write Q and H as follows

$$Q(0) = 0$$

$$Q(\bar{S} - \pi(\hat{x})\hat{q} - C(\hat{x}, y) - \ell(y) + \hat{q}) = Q(S_4) = p(\hat{x})$$

$$Q(\bar{S} - \pi(\hat{x})\hat{q} - C(\hat{x}, y)) = Q(S_3) = 1$$

and

$$H(0) = 0$$

$$H(\bar{S} - \pi(\tilde{x})q - C(\tilde{x}, \tilde{y}) - \ell(\tilde{y}) + q) = H(S_6) = p(\tilde{x})$$

$$H(\bar{S} - \pi(\tilde{x})q - C(\tilde{x}, \tilde{y})) = H(S_5) = 1$$

The assumption that both ∂q and ∂y have the same impact on the expected net loss implies

$$p(\hat{x})\,[-\ell(y) + \hat{q}] = p(\tilde{x})[-\ell(\tilde{y}) + q] \tag{23}$$

and the assumption that they are equally costly implies

$$\pi(\hat{x})\hat{q} + C(\hat{x}, y) = \pi(\tilde{x})q + C(\tilde{x}, \tilde{y}) \tag{24}$$

which implies that $S_3 = S_5$.

Moreover, using (24)

$$S_6 - S_4 = (-\ell(\tilde{y}) + q) - (-\ell(y) + \hat{q})$$

The assumption $(-x_\ell < x_q < 0)$ is equivalent here to assume that $\hat{x} > \tilde{x}$. Therefore $p(\hat{x}) < p(\tilde{x})$ and using (23) this implies that $S_6 - S_4 > 0$. We now can write $F(S) = Q(S) - H(S)$

$$F(0) = F(S_2) = 0$$

$$F(S_4) = p(\hat{x}) > 0$$

$$F(S_6) = p(\hat{x}) - p(\tilde{x}) < 0$$

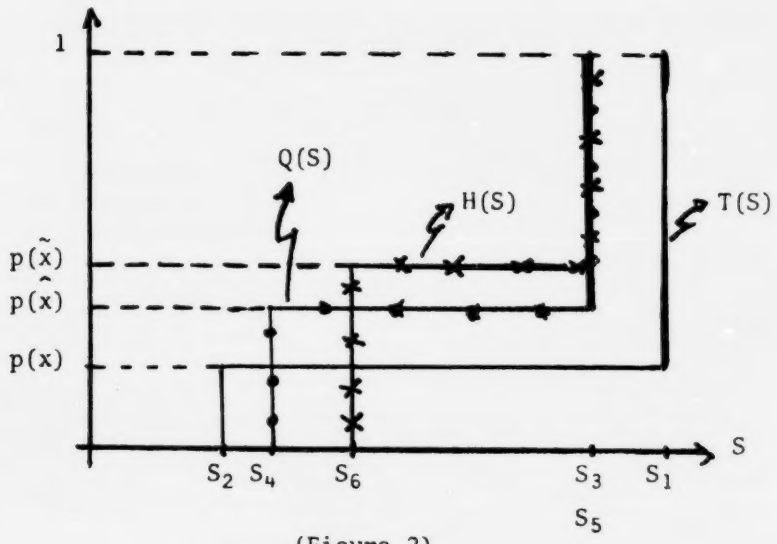
$$F(S_3) = F(S_5) = F(S_1) = 0$$

To show that F is a mean preserving spread, all we then have to show is that

$$\int_0^{S_1} F(S)dS = 0$$

But

$$\int_0^{S_1} F(S)dS = p(\hat{x})[S_6 - S_4] - (p(\tilde{x}) - p(\hat{x}))\ (S_5 - S_6)$$



(Figure 2)

$$\text{So } \int_0^{S_1} F(S)dS = p(\hat{x})\ (-\ell(\tilde{y}) + q + \ell(y) - \hat{q})$$

$$- (p(\tilde{x}) - p(\hat{x}))\ (\ell(\tilde{y}) - q)$$

$$= p(\hat{x})\ (\ell(y) - \hat{q}) - p(\tilde{x})\ (\ell(\tilde{y}) - q)$$

$$= 0 \text{ by (23).}$$

Q.E.D.

It is well known in the economic literature on insurance that if individuals are risk averse, if the insurance premium is linear and if market insurance is cheaper than self-insurance, each individual will buy full insurance coverage at the optimum, whatever his degree of risk aversion if the less risk averter buys this amount of insurance. Here we suggest a simple proof for this result and, by extension, we will have a proof for Proposition V below.

Maximizing $Z(q, x, 0)$ over q, gives as first order condition :

$$(25)\quad (1-p(x(q))\ p(x(q))[U'(\bar{S}-\pi q-\ell(0)+q\ -C(x(q),0)) - U'(\bar{S}-\pi q - C(x(q),0))] = 0$$

which implies that $q^* = \ell(0)$ at the optimum if the individual decides to take insurance. Let i stand for the less risk averse individual. Then if

$$(1-p(x_i(0))\ U_i(\bar{S} - C(x_i(0),0)) + p(x_i(0))\ U_i(\bar{S} - C(x_i(0),\ 0) - \ell(0)) <$$
$$U_i(\bar{S} - \pi q_i - C(x_i(q_i),\ 0))$$

everyone will choose full insurance coverage. Now, in order to show explicitely that this result is independent of the degree of risk aversion, one may rewrite (25), using a Taylor's expansion, and obtain :

$$(1-p(x(q))p(x(q))[U'(\bar{S} - \pi q - C(x(q),\ 0)) + U''(\bar{S} - \pi q - C(x(q),\ 0)\ (q-\ell(0))$$
$$- U'(\bar{S} - \pi q - C(x(q),\ 0))] = 0$$

or equivalently

$$(1-p(x(q))p(x(q))[U'(\bar{S} - \pi q - C(x(q),\ 0))]\left[\frac{U''(\bar{S}-\pi q-C(x(q),0))(q-\ell(0))}{U'(\bar{S}-\pi q-C(x(q),0))}\right] = 0$$

which implies that $q^* = \ell(0)$ whatever $-\frac{U''(\cdot)}{U'(\cdot)}$.

Proposition V : Assume that individuals are risk averse and that the equilibrium price of insurance is π. Assume also that the optimal quantity of insurance coverage is bounded at $\bar{q} < \ell$ in order to limit the problem of moral hazard and that market insurance is always more efficient than self-insurance. Then all risk averse individuals will buy the amount $\bar{q}$ whatever their degree of risk aversion if the less risk averter buys this amount.

Proof : Maximizing $Z(x, 0, q)$ over q such that $q \leq \bar{q}$ we obtain :

$$(26)\quad (1-p(x(q)))p(x(q))[U'(\bar{S} - \bar{\pi}q - \ell(0) + q - C(x(q),0)) - U'(\bar{S} - \bar{\pi}q - C(x(q),0))] - \lambda = 0$$

where λ is a Lagrange multiplier corresponding to $q \leq \bar{q}$.

We may rewrite (26) as :

$$(1-p(x(q)))p(x(q))[U'(\bar{S} - \bar{\pi}q - \ell(0) + q - C(x(q),0) - \bar{q} + \bar{q}) - U'(\bar{S} - \bar{\pi}q - C(x(q),0))] - \lambda = 0$$

and using Taylor's expansion

$$(1-p(x(q)))p(x(q))[U'(\bar{S} - \bar{\pi}q - \ell(0) + \bar{q} - C(x(q),0)) + U''(\bar{S} - \bar{\pi}q - \ell(0) + \bar{q} - C(x(q),0))\ (q - \bar{q}) - U'(\bar{S} - \bar{\pi}q - C(x(q),0))] - \lambda = 0$$

or

$$(1-p(x(q))\ (p(x(q))[U'(\bar{S} - \bar{\pi}q - \ell(0) + \bar{q} - C(x(q),0)) - U'(\bar{S} - \bar{\pi}q - C(x(q),0)) - \lambda]\left(\frac{1}{U'(\bar{S}-\bar{\pi}q-\ell(0)+\bar{q}-C(x(q),0))}\right) + (1-p(x(q))p(x(q))\frac{U''(\bar{S}-\bar{\pi}q-\ell(0)+\bar{q}-C(x(q),0))(q-\bar{q})}{U'(\bar{S}-\bar{\pi}q-\ell(0)+\bar{q}-C(x(q),0))} =$$

which implies by (26) that $q^* = \bar{q}$ whatever $-\frac{U''(\cdot)}{U'(\cdot)} > 0$.

Q.E.D.

Proposition VI : Under $q = \bar{q}$, risk averse individual prefer self-insurance to self-protection in terms of reduction of risk.

Proof : The proof is the same as the proof of Proposition I once $(\bar{S} - \pi\bar{q} - C(x(q), 0))$ is substituted for $\bar{S}$ and $(\ell(y) - \bar{q})$ is substituted for $\ell(y)$.

Q.E.D.

From Proposition II, it may be shown that the benefit to use self-insurance increases with risk aversion for any $\bar{q} < \ell(y)$.

We now turn to the case where individuals cannot affect the probability or the magnitude of loss but the government can.

III- Government Intervention without Individual Intervention on p and ℓ

The Government may affect the probability and magnitude of loss in different ways. For example, the Government may use fines in order to control speeding or double parking (Polinsky and Shavell [1979]) or may decide to control the inflation rate by fiscal or monetary policies (which mainly affect the probabilities of inflation rates) or by income policies (which mainly affect the magnitude or interval of inflation rates).

The model of Polinsky and Shavell [1979] which is close to the model presented in the preceding section, may be used for all the economic examples in which it is assumed implicitly that an individual cannot by himself affect the probability or the magnitude of loss. But for the discussion we shall limit ourselves to the example of double parking.

Polinsky and Shavell have shown that the risk behavior of the agents is important to explain how to establish the optimal levels of probability and magnitude of fines. In fact, what they show explicitly is that the optimal tradeoff is different under risk neutrality than it is under risk aversion. But they don't show explicitly how risk aversion affects the results. In this part of the paper, we will show that their result under risk neutrality, depends essentially on the different costs of applying both policies. We will show that if it is costless to catch individuals (or if the cost functions of maintaining p and of establishing the fine are the same), risk neutral individuals would be indifferent between any combination of the magnitude and the probability of fine which insure the same expected tax return as the amount corresponding to the combination of the threshold probability and the individual's wealth. Moreover we will show how 1) risk aversion affect explicitly the optimal tradeoff and 2) how different degrees of risk aversion implies different tradeoffs. Finally we will present a method to calculate the compensation to the more risk averse group if the less risk averse group decides for the society.

IIIA) The Polinsky-Shavell's Model

There are two possible gains from engaging in double parking, v and w where $w > v$. Let $\rho(0 < \rho < 1)$ be the probability that the gain will be v. P is the insurance premium against external cost and it is equal to ne where n is the proportion of the population who engage in double parking and e the external cost ($v < e < w$ by assumption). ℓ now corresponds to the fine and p is the probability that an individual who engages in double parking,

is caught. The Government uses taxes and fines to finance the cost of maintaining p. Therefore $t = c(p, \lambda) - np\ell$ where t means per capita taxes, $c(p, \lambda)$ is per capita cost of maintaining p and $np\ell$ is the revenue from fines. λ is a shift parameter of the cost function; $c_\lambda > 0$.

An individual will double park if

$$(27) \qquad Z(v) \equiv (1-p)U(\bar{S} - t - P + v) + pU(\bar{S} - t - P + v - \ell) > U(\bar{S} - t - P)$$

or if

$$(28) \qquad Z(w) \equiv (1-p)U(\bar{S} - t - P + w) + pU(\bar{S} - t - P + w - \ell) > U(\bar{S} - t - P)$$

where (27) is for the gain v and (28) is for the gain w. (27) and (28) determine the value of n.

Now let's assume, for the moment, that there is only one class of risk behavior in this economy. Since now all individuals are identical, the problem of the determination of p and ℓ is the problem of maximizing the expected utility of one individual (EZ) which is equal to (before the individual knows whether he will gain v or w) :

$$(29) \qquad \rho Z(v) + (1-\rho)Z(w)$$

Finally the authors use the concept of threshold probability ($\bar{p}$) defined as the probability which is such that the individual would be indifferent between using double parking or not if the fine is his wealth. As example if the gain is w and the fine is $(\bar{S} - t - P)$, $\bar{p}(\bar{S} - t - P, w)$ is such that :

(30) $(1-\bar{p}(\bar{S} - t - P, w))U(\bar{S} - t - P + w) + \bar{p}(\bar{S} - t - P, w)\ U(w) = U(\bar{S} - t - P)$

where under risk neutrality $\bar{p}(\bar{S} - t - P, w) = \frac{w}{\bar{S} - t - P}$

IIIB) Risk Neutral Individuals

Proposition VII : Assume that it is optimal to control double parking and that the individuals are risk neutral. Assume also that it is costless to catch individuals who engage in double parking. Then the risk neutral individuals are indifferent between any combination of p (greater than the threshold probability of those who gain the least) and ℓ (smaller than the individual's wealth) which corresponds to the same expected taxes revenues as the combination corresponding to the threshold probability (of those who gain the least) and wealth.

Remark : This proposition is also true if the cost's structures of maintaining and of establishing the fine are the same. This proposition can be proved in two ways. One is to use the approach of the preceding section and to show that the risk neutral individual is indifferent between any mean preserving spread corresponding to $[\bar{p}(\bar{S} - t - P, v)]\ [\bar{S} - t - P]$. This way will not be presented here. The other one is to adapt the proof of Polinsky and Shavell to the situation in which it is costless to catch individual. We will only present the different points here.

Proof : Since $v < e < w$, from the proof of Polinsky and Shavell we know that it is optimal to have $p^* = \bar{p}(\bar{S} - t - P, v)$ and, by definition of the threshold probability, $\ell^* = \bar{S} - t - P$. Therefore the expected fine is equal to $p^*\ell^*$ and the per capita tax is equal to $-(1 - \rho)p^*\ell^*$. Now let's assume that $\hat{p} > \bar{p}(\bar{S} - t - P, v)$ and that $\hat{\ell} < (\bar{S} - t - P)$ such that

$\hat{p}\,\hat{\ell} = [\bar{p}(\bar{S} - t - P, v)]\,(\bar{S} - t - P)$. At $\hat{p}$ and $\hat{\ell}$ so defined, $\hat{p}\,\hat{\ell} \geq v$ and $\hat{p}\,\hat{\ell} \leq w$ that is to say, only the individual gaining w would engage in double parking. Therefore P would not change and the taxes would not change. Since nothing changes, expected utility does not change which is enough to proof the Proposition since this is true for any combination of p and ℓ such that $p\,\ell = [\bar{p}(\bar{S} - t - P, v)]\,[\bar{S} - t - P]$.

Q.E.D.

IIIC) Risk Averse Individuals

Proposition VIII : Assume that it is optimal to control double parking and that the individuals are risk averse. Then, if it is costless to catch individuals, the optimal probability is equal to one and the optimal fine is equal to w (the private gain of those who engage in double parking).

Remark I : Here we have to be careful. The question is not the following : for a given proportional (exogenously) increase in p or in ℓ, which one is the more efficient to reduce double parking? Using the same methodology as in the previous section of the paper, we may answer to the preceding question in showing that a proportional increase in fine is more efficient under risk aversion than a proportional increase in p, since it reduces more the expected utility associated to double parking[1]. Here the question is : what is the best combination of p and ℓ in utility terms, for a risk averse consumer who represents the society (since all individuals are identical)?

[1]See Boyer and Dionne [1982] for more details.

Remark II : Proposition 2 of Polinsky and Shavell says that "if the cost of catching them (risk averse individuals) is sufficiently low, the optimal probability equals one and the optimal fine equals the private gain of those who engage in the externality creating activity" (p. 884). Here there is a tradeoff between the reduction of risk and the cost of catching individuals and the probability will be less than one only if the cost structure is sufficiently high to reverse the benefits in terms of reduction of risk associated to a probability equal to one[1].

Remark III : The first part of the proof of Polinsky and Shavell shows that, when it is costless to catch risk averse individuals, the optimal probability is one and the optimal fine is w. Here we give a different proof of that result by showing that for any mean preserving spread of Polinsky and Shavell's result, the expected utility of the risk averse individual is reduced. This simple proof will be useful to prove the next proposition on the relation between the variation of risk aversion and the optimal tradeoff between p and ℓ.

Proof : With $\ell = w$ and $p = 1$, the expected utility of the representative consumer is $U(\bar{S} + (1 - \rho)(w - e))$. Now let's choose a $\hat{p} < 1$ and a $\hat{\ell} > w$ such that $\hat{p}\,\hat{\ell} = w$.

[1]Or using Polinsky and Shavell's terminology "this is because the disutility from bearing the risk of being caught is more important than the savings in the cost of catching individuals", p. 884.

We have to compare :

$$(31)\quad U(\bar{S} + (1 - \rho)\ (w - e)) \gtreqless \rho\ U(\bar{S} + (1 - \rho)\ (\hat{p}\ \hat{\ell}\ - e))$$
$$+\ (1 - \rho)[(1 - \hat{p})\ U(\bar{S} + (1 - \rho)(\hat{p}\ \hat{\ell} - e) + w)$$
$$+\ \hat{p}\ U(\bar{S} + (1 - \rho)\ (\hat{p}\ \hat{\ell}\ - e) + w - \hat{\ell})]$$

Taking Taylor's series expansion on the right hand side and writing w for $\hat{p}\ \hat{\ell}$

$$(32)\quad U(\bar{S} + (1 - \rho)\ (w - e)) \gtreqless \rho\ U(\bar{S} + (1 - \rho)\ (w - e))$$
$$+ (1 - \rho)[(1 - \hat{p})\ U(\bar{S} + (1 - \rho)\ (w - e)) + (1 - \hat{p})\ U'(\bar{S} +$$
$$(1 - \rho)\ (w - e))w + (1 - \hat{p})\ U''(\bar{S} + (1 - \rho)\ (w - e))\ \frac{w^2}{2} +$$
$$\hat{p}\ U(\bar{S} + (1 - \rho)\ (w - e) + \hat{p}\ U'(\bar{S} + (1 - \rho)\ (w - e))\ (w - \hat{\ell})$$
$$+ \hat{p}\ U''(\bar{S} + (1 - \rho)\ (w - e))\ \frac{(w - \hat{\ell})^2}{2}\ ]$$

which implies that

$$(33)\quad U(\bar{S} + (1-\rho)\ (w-e)) > U(\bar{S} + (1-\rho)\ (w-e)) + U''(\bar{S} + (1-\rho)\ (w-e))\ (\tfrac{1}{2}(w^2 + w\hat{\ell}\ - 2w^2))$$

under risk aversion since $\hat{\ell}\ > w$ and $U''(\cdot) < 0$

Q.E.D.

Proposition IX : Assume that it is optimal to control double parking and that the individuals are risk averse. Assume also that it is costless to catch individuals. Then the disutility, associated to any combination of p and ℓ representing a mean preserving spread from $p^* = 1$ and $\ell^* = b$, increases with the degree of risk aversion.

Proof : The proof is straightforward from the relation (33) of the preceding proof. Dividing both sides by $U'(\bar{S} + (1 - \rho)\ (w - e))$ gives

$$(34)\quad \frac{U(\bar{S}+(1-\rho)(w-e))}{U'(\bar{S}+(1-\rho)(w-e))} > \frac{U(\bar{S}+(1-\rho)(w-e))}{U'(\bar{S}+(1-\rho)(w-e))} + \frac{U''(\bar{S}+(1-\rho)(w-e))\frac{1}{2}(w^2+w\hat{\ell}-2w^2)}{U'(\bar{S}+(1-\rho)(w-e))}$$

Since the ratio $-\frac{U''(\cdot)}{U'(\cdot)}$ increases with risk aversion, the differences between the left hand side and the right hand side increases with risk aversion[1].

Q.E.D.

Now if we define λ_k^* as the highest λ [2] that permits the optimal tradeoff ($p_k^* = 1$, $\ell_k^* = w$) to be chosen, therefore, we can say from Proposition 2 of Polinsky and Shavell and from Proposition VIII of this paper, that λ_k^* increases with risk aversion that is $\lambda_i^* < \lambda_j^*$ if $-\frac{U_i''(\cdot)}{U_i'(\cdot)} < -\frac{U_j''(\cdot)}{U_j'(\cdot)}$ for all S.

IIID) Compensation for the More Risk Averse Individuals

Let's assume that there are two groups of individuals in the economy and that there are more type i individuals than there are type j individuals. Let's also assume that the less risk averters (k = i), for a given λ sufficiently high, prefer a tradeoff corresponding to a $p_i^* < 1$ and a $\ell_i^* > w$. However those who are more risk averters (k = j) prefer a tradeoff corresponding to a $p_j^* = 1$ and a $\ell_j^* = w$ for the same λ. This situation is clearly possible from the analysis of the preceding section.

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[1]This proposition is a special case of Proposition II.

[2]As mentioned above, λ is a shift parameter of the cost function; $C_\lambda > 0$.

Using majority rule, the tradeoff of the society is chosen by type i individuals. In order to consider only the effects of risk variation, let's assume that the type j individuals still engage in double parking under p_i^* and ℓ_i^* that is (for the gain w only) :

$$(35) \quad U_j(\bar{S} + (1-\rho)\ (w-e) - C(p_i^*,\lambda)) < (1-p_i^*)U_j(\bar{S} + (1-\rho)\ (w-e) - C(p_i^*,\lambda) + w) + p_i^* U_j(\bar{S} + (1-\rho)\ (w-e) - C(p_i^*,\lambda) + w - \ell_i^*)$$

where it is implicitly assumed that $\rho_j = \rho_i = \rho$ and that $p_i^* \ell_i^* = w$. These assumptions are necessary in order to isolate only the effect of risk variation and to calculate the compensation associated to this variation. Therefore the expected wealth is the same for both groups.

Under these assumptions the optimal compensation (m) which would make individuals j indifferent between both policies is given by :

$$(36) \quad U_j(\bar{S} + (1-\rho)\ (w-e) - C(1,\lambda) - m) = \rho U_j(\bar{S} + (1-\rho)\ (w-e) - C(p_i^*,\lambda)) + (1-\rho)\ [(1-p_i^*)U_j(\bar{S} + (1-\rho)\ (w-e) - C(p_i^*,\lambda) + w) + p_i^* U_j(\bar{S} + (1-\rho)\ (w-e) - C(p_i^*,\lambda) + w - \ell_i^*)]$$

which implies,

$$m = -\frac{U_j''(\cdot)}{U_j'(\cdot)}\ (1/2(w^2 + w\,\ell_i^* - 2w^2)) + [C(p_i^*, \lambda) - C(1, \lambda)]$$

under risk aversion. Moreover m increases with risk aversion. The first term is the compensation associated with the variation of risk and the second term is the compensation due to the difference in costs of maintaining p.

IV- Individuals' Activities and Government Intervention

In this section we present a general model incorporating parts of both models of the preceding sections. Auto safety regulation is a good example of government intervention aimed at reducing the social costs of accidents since individuals do not consider externalities in their private activities. There is, for the society, an optimal choice of accident probability and severity. Generally safety devices are designed to decrease the loss from an accident. However the government may decide to reduce the probability of accidents by reducing speed limits (which may also reduce the loss from a given accident) or by taking actions against drivers under the influence of alcohol[1]. Those government actions have ambiguous effects on private activities (Peltzman [1975]) : they can increase or decrease the private incentives. For example, Peltzman argues that an increase in the use of mandatory safety equipment may decrease incentives for safe driving and then increase the probability of accidents or at least of some types of accidents.

Here, contrary to the model presented in the preceding section, we are not concerned with the optimal choice between the probability of accident and the loss associated to an accident. We use instead the model of Peltzman, under risk aversion, and show that, for a given mean preserving spread between the variation of the probability of an accident and the variation of the amount of loss, risk averse individuals may prefer, be indifferent or not prefer the policy that affects the

[1]On auto safety regulation see Arnould and Grabowski [1981], Oi [1973], and Peltzman [1975].

amount of loss. This ambiguous result is explained by the fact that now individuals may react to government policies by taking private actions which may have opposite effects on the variation of risk.

IVA) The Model

Let's assume that all the individuals have the same risk aversion. As in section II, an individual may change the probability of loss and the amount of loss by his activities. However, the Government may also affect those magnitudes by his activities a and b; therefore p is now function of x and b and ℓ is function of y and a where $p_b(x,b) < 0$, $p_{bb}(x,b) > 0$, $\ell_a(y,a) < 0$, $\ell_{aa}(y,a) > 0$ and $\ell_{ay} = p_{xb} = 0$.

Market insurance is available and is more efficient than self-insurance but, since moral hazard is potentially present, $\bar{q} < \ell$ and, therefore, self-insurance and self-protection are used. Under these assumptions, the level of expected utility ($Z(x, y, \bar{q}, a, b)$) for a given $\bar{q}$ is equal to :

$$(37) \quad (1-p(x,b))U(\bar{S} - \bar{\pi}\bar{q}) + p(x,b)U(\bar{S} - \bar{\pi}\bar{q} - \ell(y,a) + \bar{q}) - C(x,y)$$

where we assume that government actions are costless and do not affect individual costs directly. Moreover we assume that the costs of x and y are in utility terms in order to simplify the presentation. For given values of a and b, the chosen amounts of x and y are given by :

$$(38.a) \quad Z_x \equiv p_x[U(2) - U(1)] - C_x = 0$$

$$(38.b) \qquad Z_y = - p(\cdot)\, \ell_y\, U'(2) - C_y = 0$$

where $U(1) = U(\bar{S} - \bar{\bar{\pi}}\bar{\bar{q}})$

$U(2) = U(\bar{S} - \bar{\bar{\pi}}\bar{\bar{q}} - \ell(y,a) + \bar{q})$

$p(\cdot) = p(x,b)$

Second order conditions are

$$Z_{xx} = p_{xx}[U(2) - U(1)] - C_{xx} < 0$$

$$Z_{yy} = - p(\cdot)\, \ell_{yy} U'(2) - p(\cdot) U''(2) \ell_y - C_{yy} < 0$$

$$\Delta = \begin{vmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{vmatrix} > 0$$

where $Z_{xy} \equiv - p_x U'(2) \ell_y < 0$

The government may affect these chosen amounts of x and y by its choice of a and b. Totally differentiating (38) and (39) and using

$$Z_{xa} \equiv - p_x U'(2) \ell_a < 0$$

$$Z_{ya} \equiv + p(\cdot) U''(2) \ell_y \ell_a < 0$$

$$Z_{xb} = 0$$

$$Z_{yb} \equiv - p_b\, \ell_y U'(2) < 0$$

We can derive using Cramer's rule,

$$(39) \qquad \left.\frac{\partial x}{\partial a}\right|_{\partial b = 0} = \frac{1}{\Delta}\{- Z_{xa} Z_{yy} + Z_{xy} Z_{ya}\} \gtreqless 0$$

$$(40) \qquad \left.\frac{\partial y}{\partial a}\right|_{\partial b = 0} = \frac{1}{\Delta}\{- Z_{xx} Z_{ya} + Z_{xa} Z_{yx}\} \gtreqless 0$$

$$(41) \qquad \left.\frac{\partial x}{\partial b}\right|_{\partial a = 0} = \frac{1}{\Delta}\{+ Z_{xy} Z_{yb}\} > 0$$

$$(42) \qquad \left.\frac{\partial y}{\partial b}\right|_{\partial a = 0} = \frac{1}{\Delta}\{- Z_{xx} Z_{yb}\} < 0$$

VIB) The effect of the Government's intervention on the variation of risk

Proposition X : Assume that the average loss is affected by the same amount from Government policies a and b. Assume also that these policies are costless for the individuals and that individuals are in equilibrium in x and y for a given $\bar{q} < \ell$. Then a risk averse individual prefers the Government policy (a) to Government policy (b) if and only if the product of the total direct effects of the policies is not offset by the product of their cross-effects, that is

$$(43) \qquad \left(\frac{dp}{db}\right)\left(\frac{d\ell}{da}\right) > \left(\frac{dp}{da}\right)\left(\frac{d\ell}{db}\right) .$$

Proof : From (39)-(42) we have that the expected loss (μ) depends on a and b,

$$\mu = p(x(a,b),\ b)\ \ell(y(a,b),\ a)$$

and its variations from changes in policies a and b are

$$(44) \qquad \partial\mu = \{p_b\, \ell(\cdot) + p_x\, x_b\, \ell(\cdot) + p(\cdot)\, \ell_y\, y_b\}\, \partial b$$

$$(45) \qquad \partial\mu = \{p(\cdot)\, \ell_a + p_x\, x_a\, \ell(\cdot) + p(\cdot)\, \ell_y\, y_a\}\, \partial a$$

Taking into account the possible reaction of the individuals to the Government policies, the equilibrium level of expected utility for given $\bar{q}$, a, b is equal to :

$$(46) \quad (1-p(x(a,b), b)\, U(S - \pi\bar{q}) + p(x(a,b), b)\, U(\bar{S} - \pi\bar{q} - \ell(y(a,b), a) + \bar{q}) - C(x(a,b), y(a,b))$$

and the total effects of Government interventions on utility are :

$$(47) \quad \partial Z = \{p_b\, [U(2) - U(1)] + p_x\, x_b\, [U(2) - U(1)] - p(\cdot)\, U'(2)\, \ell_y\, y_b - C_x x_b - C_y y_b\}\, \partial b$$

$$(48) \quad \partial Z = \{-\, p(\cdot)\, U'(2)\, \ell_a - p(\cdot)\, U'(2)\, \ell_y\, y_a + p_x\, x_a [U(2) - U(1)] - C_x\, x_a - C_y\, y_a\}\, \partial a$$

Using the assumptions on the total variation of the expected loss (44) = (45) and on the total variation of costs in utility terms, the comparison of (47) and of (48) is reduced to compare :

$$(49) \quad dZ = \{p_b[U(2) - U(1)] + p_x\, x_b\, [U(2) - U(1)] - p(\cdot)U'(2)\ell_y y_b\}\{\frac{A}{B}\}\, \partial a = C\, \partial\alpha$$

$$(50) \quad dZ = \{-\, p(.)U'(2)\ell_a - p(.)U'(2)\ell_y y_a + p_x\, x_a\, [U(2) - U(1)]\}\, \partial a = D\, \partial a$$

where $A \equiv p(\cdot)\, \ell_a + p_x\, x_a\, \ell(\cdot) + p(\cdot)\, \ell_y\, y_a$

$B \equiv p_b\, \ell(\cdot) + p_x\, x_b\, \ell(\cdot) + p(\cdot)\, \ell_y\, y_b$

Policy ∂a will be prefered to the equivalent policy $\partial b = \frac{A}{B}\, \partial a$ if and only if, from (49) and (50),

(51) $C < D$.

If we assume that $x_b = y_b = y_a = x_a = 0$, from Proposition I we know that policy a is prefered to policy b in terms of reduction of risks. But since the individuals may react here to the government policies and since both types of reactions may have opposite effects, then the comparison of policies, in utility terms, is ambiguous, that is C may be larger than, equal to or smaller than D.

After some algebric manipulations, (51) can be rewritten

$$Y[U(2) - U(1)] > -Y\, U'(2)\ell(.)$$

where $Y \equiv [(p_b + p_x\, x_b)\, (\ell_a + \ell_y\, y_a) - p_x\, x_a\, \ell_y\, y_b]$

That is ∂a is prefered to ∂b if and only if from (8), $Y > 0$. But $Y > 0$ if and only if (43) holds.

Q.E.D.

However when the individuals can only affect either x or y instead of both activities at the same time, we have

a) when $y \equiv 0$, $\ell(\cdot) = \ell(a)$ and $p(\cdot) = p(x,b)$. Then $x_a < 0$, $x_b = 0$ and $Y = p_b\, p(\cdot)\ell_a > 0$. So ∂a is always prefered to ∂b.

b) when $x \equiv 0$, $\ell(\cdot) = \ell(y,a)$ and $p(\cdot) = p(b)$. Then $y_a < 0$, $y_b < 0$ and $Y > 0$ if and only if $\left(\frac{d\ell}{da}\right) = \ell_a + \ell_y\, y_a < 0$.

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